The Arithmetic Teacher

DECEMBER • 1954

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A Quarterly Journal of
THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

THE ARITHMETIC TEACHER

Published Quarterly by
The National Council of Teachers of Mathematics

Editor: BEN A. SUELTZ, State University Teachers College, Cortland, N. Y.

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Subscription Price: $1.50 per year (four issues) to individual subscribers, $2.50 per year to others (libraries, schools, colleges, etc.)

All editorial correspondence, including books for review, should be addressed to the Editor. Advertising correspondence, subscriptions to THE ARITHMETIC TEACHER, and notice of change of address should be sent to:

The National Council of Teachers of Mathematics
1201 Sixteenth Street, N.W., Washington, D.C.

Entered as second-class matter at the post office at Washington, D. C., June 21, 1954, under act of March 3, 1879. Additional entry granted at Menasha, Wisconsin.

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THE ARITHMETIC TEACHER

Volume I — Number 4

December — 1954

Unifying Ideas in Arithmetic

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In our discussion of unifying ideas in arithmetic, it makes a difference whether we consider the first word of our topic as a qualifying adjective or as the participial form of a verb of action—that is, whether we determine that our subject is ideas, or instead human individuals who may sometimes have ideas and do things with them. In the one case, we tempt ourselves to treat ideas as though they stand alone and, under proper circumstances, unite themselves; in the other, we keep away from the futile effort to abstract ideas from the human minds that conceive them. I choose the latter alternative. I prefer to think of the pupil as an active organized personality who can operate under his own power in bringing his ideas into patterns of unity. I shall not thereby neglect the characteristic unifying qualities which ideas in arithmetic possess.

Making a Start

The pupil may start unifying his ideas in arithmetic at the beginning of his school work. If his teacher inquires, "How many books are on the table?" he counts to find out. He then tells his answer: *three*, let us say. At the same time, he may write the figure *3*, as a way to tell his answer, with pencil or chalk. Though he has some difficulty writing the figure, he has none with the idea of its use. Writing the figure, as another way to tell his idea of the group, is none the less simpler than writing the word *George*, as another way to tell his name. It is never a mystery, except as the teacher keeps it hidden and makes it so.

A bit later, the beginning pupil may bring into a unity of thought and action the idea of a question to answer, and the writing of the expression that tells the answer. Thus he may look at the figures *2* and *3* in a column, $\begin{array}{r}2\\ \underline{3}\end{array}$, as the question, "Two and three are how many?" When he finds the answer, he is ready to tell it, either orally or by completing or producing the column of *2* and *3*, with *5* underneath, $\begin{array}{r}2\\ \underline{3}\\ 5\end{array}$. His written statement, $\begin{array}{r}2\\ \underline{3}\\ 5\end{array}$, tells his answer equally as well as his oral statement, "Two and three are five." In this procedure, he may move with certain confidence; for he goes forward in his thinking, directly from a question that calls for an answer, to the answer he finds and has in mind to tell. The procedure is not the usual memoriter learning of the teacher's statement, or the book's statement, of a fact—in this case, a so-called "addition fact." The procedure here, though outwardly much the same, is a different one. Instead of a "fact" to learn, the pupil has a question to answer. Instead of a statement to copy, the pupil gets an idea of something to seek and find. The succeeding writing is simply the coincidence of telling what he knows when he knows it.

Contradictions in Practice

Our two illustrations of the way the pupil, at the very beginning of his work in arithmetic, may go about unifying his ideas has abundant contradiction in the common outcomes of the usual arithmetic program in the primary grades. Thousands of pupils seldom get a chance to write the numerals as expressions of ideas until they have been in school one or two years; and often then only in roundabout relation to ideas, through the words and sentences which originally have had the exclusive agency of expression. Many pupils, during the whole of their lives, at least of their school lives, never quite succeed in making numerical expressions do more than record the results of their thinking, after they somehow have done it. They never quite get around to the idea of using their numerical expressions to facilitate the combining of ideas and to aid their thinking. In the case of some, the simple notation of the nine numerals and zero in a few familiar forms is always a mysterious rite.

Yet the contradictions the large numbers of our pupils provide are only apparent contradictions. Of course there appears before many pupils, between the ideas they think and say and the expressions they must write, an impenetrable wall. But it is we, their teachers, who have built the wall. We cultivate the illusion of what we call "concrete arithmetic," which we think our pupils should understand; and then we distinguish it from an imaginary "abstract arithmetic," which, we can be sure, no one can understand. Thereby we deprive our beginning pupils of the aid to their thinking which its written expression could give; and then we remove the written expression so far from the original thinking of our pupils as to make it difficult in the extreme for them ever to bring it back to reality. For one thing, we delay our beginners, who are ready and eager to move ahead, with a lot of "readiness" activity they do not need; and, for another, we delay, often for a year or more, the writing of what little they do manage to think. The inevitable consequence is that the beginning pupils, who do manage to unify their ideas in arithmetic, are few and far between. The thing of wonder is that so many are able to gain any ideas at all. The contradictions of the possibility of unifying ideas are the contradictions which, through our procrastinating, we take pains to create.

Proper Timing

Yet not all our timing of the pupil's progress in arithmetic is thus so bad. In illustration, we may consider the practice of many teachers of delaying the introduction of terms, until the pupil has at least the essence of the ideas the terms are useful in designating. Thus, when the pupil has found answers over and over to such questions as "Two and three are how many?" $\begin{array}{r}2\\ \underline{3}\end{array}$ and, through repeated demonstration, has become conscious of the way to find answers, his teacher supplies the name "addition" for his activity. How to explain the term, in such case, is not a problem; for the pupil already has supplied his own explanation. The teacher has merely to provide the name for what the pupil is sure he knows. Though his knowledge of addition has just begun, he can expand his new term as he enlarges his idea. Moreover, his new term adds, or gives new emphasis, to his enlarging idea.

The whole procedure in its proper timing relieves the teacher of a great, and somewhat uncertain, responsibility. Of late, teachers have been hearing a lot about "meaningful arithmetic," including suggestions on how they should make arithmetic "meaningful" for their pupils. They have had to contemplate discussions of what the meaning actually is, which they are called upon to transmit. Since all such definitions of the meaning of meaning require further definitions of the defining terms, teachers who try to pursue them become lost in the maze of philosophic erudition, or just plain verbalism. Their only relief is a return to the basic fact that

the only meaning in arithmetic, that means anything to the pupil, is the meaning he provides himself through the activity of his own thinking. In illustration, we may consider how the pupil may build his idea of *division*, for example, and bring it into use as a means of unifying other ideas he gains as he moves along.

Let us see what the pupil may make his dividing mean.

Dividing as Illustration

We may seek in the usual way an answer to the kind of question we usually raise, namely, "How does the pupil acquire the *meaning* of *division?*" But that is not the question we should ask. Such a question suggests that division is one thing the pupil learns; its *meaning*, something more. The question we should ask is rather: "What does the pupil make his dividing—the dividing that *he* does—mean to him?" If we look for an answer to a question like this, we find that dividing means to the pupil just what *he* does and thinks as he divides, no more and no less.

We find something else, if we pursue a question like the latter, that is a cardinal advantage. We find that we can maintain control of the pupil's work and thinking as he does his dividing, that we can guide the pupil's attention as he works and thinks.

The pupil divides to answer a question: "How many twos are in eight?" $2\overline{)8}$. We raise the question and we make the question clear. We see that the pupil pays attention to three things: (1) the total; (2) the equal groups; (3) the number of the equal groups. The pupil does three things, giving each careful attention: (1) He counts out eight and checks for certainty; (2) He counts out a two, then another, and another, and another, and checks to be sure that each is a two; (3) He counts the twos, and counts again to make sure. Now the pupil has his answer—*his* because *he* has found it—and he tells and writes it,

"Four twos are eight," $\begin{array}{r}4\\2\overline{)8}\\\underline{8}\end{array}$

Somewhat similarly, the pupil gives attention to three things when he answers the question, "How much is one half of eight?" (1) He pays attention to the total original amount. (2) He makes sure oi the equality of the parts. (3) He finds the number in one of the equal parts.

Steps in Dividing

The pupil's activity of paying attention to the division question and the three things he thinks and does when he divides, with its succeeding and expanding repetitions, is the *first* step in his learning about dividing. The question for which he seeks an answer, the kind of answer he should seek, the dividing he does, and the objects of his attention throughout, are all in the simplest possible setting. Such a setting has its concrete, or objective, features reduced to the necessary minimum.

The pupil may now move to a *second* and a *third* step in dividing. In taking each of these, he continues to give attention as he did in taking the first: to the original whole amount; to the equality of the groups, or parts; to the number of equal groups, or the number on each equal part. Thus, basically, there is no difference in the three steps; the only existing difference is in their outward appearance, or in the manner in which the pupil's division question is made to appear. In the first step, the question appears, as it were, in a state of nature, unclothed in any garment that diverts the attention; and in the second and third steps, the question comes clothed in the dress of a practical situation: the familiar everyday clothes, in the one; and the somewhat unfamiliar Sunday's best, in the other. To illustrate,

Second step. The pupil deals with "problems" that familiar situations provide: "Helen earned 75¢ baby-sitting. She saved one-third of what she earned. How much did Helen save?"

Third step. The pupil deals with "problems" requiring study of new and somewhat unfamiliar situations: "Jack and Don had a lemonade stand. Jack furnished one-third of the materials. The profit, after they had paid

all expenses, was 90¢. How much was Jack's share?" In this "problem," the matters of "profit, after expenses" and proper "wages" to the two boys, complicate the division question, and must be studied and cleared to make the question clear.

Dividing as a Unifying Idea

The pupil can make his idea of dividing into equal parts a unifying idea that runs through and clarifies much of his later studies, such as fractions, measures, and decimals. He may note and recognize the common features of his working and thinking when he divides a whole into its equal parts, when he divides a measure into its equal smaller measures, when he divides into tenths and tenths of tenths, and when he deals with decimal divisions as measures.

Similarly, the pupil can make his idea of dividing into equal groups a unifying and clarifying idea as he seeks to learn the later and more complex processes, such as dividing by a two-place number. His earlier attention to total original amount, equality of the separated groups, and number in each, helps him to keep clear what the question is and what the answer should be that he must seek. Similarly also, the same clear question, and clear idea of answer to seek, bring the difficult idea of dividing by fractions out into the open, where it too can be clear. Any pupil who recognizes the example, $4 \div \frac{1}{2} =$, as the question, "How many halves are in four?" recognizes no difficulty in the task he must perform.

The pupil's meaning of dividing, as being what he thinks and does as he divides, is but an illustration of the meaning he may make of all the thinking processes he learns to perform, whether those of adding, or subtracting, or multiplying, or computing the average, or finding per cent. If he works, from the outset, toward the simpler answers his simpler questions suggest, he may gain an idea of procedure which may carry him along, when both questions and answers become complicated in newer requirements and usages. And if he attends closely to his idea, he may recognize it as a common thread running through many phases of his later work, and uniting them into a single pattern.

Our major concern is that the pupil must give his continued attention to the central, or characteristic, feature of an arithmetical process as he learns to put it into operation; or, conversely, that we must do nothing to distract his attention and draw it away from the central, or characteristic, feature.

Distractions We Make

As teachers, we seem always to go out of our way to distract the attention of pupils from the main business in which they should engage in learning arithmetic.

We no sooner introduce the pupil to the way he should work and think, as he starts learning a process, like dividing, than we begin to overwhelm him with all sorts of personal and social situations, such as divisions of his time and money, of his friends into groups for work and games, and of his property into shares; and with objects of all kinds, and shapes, and colors, in all sorts of designs, that both attract and distract. We call this the pupil's "concrete arithmetic." We properly recognize that, to begin his studies of grouping, the pupil at the outset must have objects to group; but we tend to forget that it is the *grouping* he must study, not the qualities of his objects. The paraphernalia of present-day primary arithmetic is out of all proportion to any possible arithmetical usage pupils can make of it.

That isn't all. Not content with distracting the pupil's attention from the characteristic features of the grouping he should study and learn to diverting situations and interesting concrete materials, we go further and set apart in isolation, as something "meaningless" and "abstract," such computational result as the pupil may incidentally gain. Of course it is "meaningless." There is no meaning for the pupil except that which he creates. Of

course it is "abstract." It stands so far apart from everything that makes sense for the pupil that he seldom can connect it sensibly with anything. Computation and problem solving have always been widely-separated activities in the arithmetic of the school. We do very little in present-day teaching to get our pupils able to recognize them as a single thinking procedure in different dresses.

Yet when we return to our illustration of the pupil creating his meaning of dividing, for example, by paying attention to what his division question asks, and to the kind of answer he should seek, we can see how the meaning he thus creates is a common characteristic in all three stages of the dividing he must learn to perform. Thus, in the first stage, the pupil meets his question and its suggestion as to the kind of answer, unadorned and void of distracting surroundings. In the second and third stages, he may recognize and attend to the same question, and suggested kind of answer, when they are concealed in practical situations, familiar and to be learned, respectively. He thus may make his idea of question and suggested answer an idea that unifies his computations and his problem exercises.

Two Ways of Unifying Ideas

To this point we have considered the matter of the pupil unifying his ideas in arithmetic in two ways. The two ways are not sharply distinguishable, for in both the pupil must first gain ideas to be able to unify them, and the pupil is the unifying agent.

In one of the two ways, we have seen the pupil unifying ideas that normally belong together, except as someone works to keep them apart. Thus, in our illustrations, we have seen how the pupil may unify his idea of a quantity with his act of writing the proper figure to tell what it is, and, in similar manner, how he may unify his idea of a number-question and its answer with the act of writing the figures in proper form as a means of expression. We have seen, further, how the pupil may unite a process, when he has learned it, with its proper name; and how he may unify his computation and his problem solving into a single requirement of practice in thinking.

In the other of the two ways, we have seen how the pupil may use an idea, such as that of dividing into equal parts, as a means of unifying into a common pattern of work and thinking the otherwise diverse studies of fractions, of divisions of measures, of divisions into tenths and tenths of tenths, and of decimals as measures.

In further illustration, we may consider briefly how the pupil may make his idea of "dealing with tens" as he "deals with ones" a clarifying as well as unifying idea.

Here again, whether the pupil gains the unifying idea, and uses it in unifying, depends upon the way he thinks as he works. Here again, it is a case of the thinking making the working worth while.

Ten as a Unifying Idea

Everyone must deal with tens, and multiples of ten, as he deals with ones. There is no other way to deal with them. But whether he is conscious of the fact is the point of difference.

When the child first can count to two or three, or four or five he counts everything in the range of his senses: chairs in the room, pictures on the wall, plates on the table, trees in the yard. Later, as a pupil, he counts rows and bunches and piles and bundles of things the way he counts ungrouped objects. So, when he provides himself a few groups of tens, he can, and does, count them as he counts ones. And he counts separated groups together as he counts separated objects, and thus he demonstrates, usually without knowing it, that he adds tens the way he adds ones. It is at this point, if not also long before, that the teacher can become a guiding light, making the pupil conscious of what he does.

Thus, having learned to add 2 and 3, the pupil finally faces the task of adding

20 and 30, 2 tens and 3 tens. Yet, as a task, the new learning is of less difficulty, provided that the pupil knows and recognizes that he now has *tens* to add. This he can do, and does, in the degree that his earlier attention to the special positional value for writing tens, and his teacher's present question about how he should add tens, make him conscious of the special thinking he must do.

As the pupil moves ahead, let us say that his teacher continues to call his attention to the way he deals with *tens:* How do we subtract tens? How do we multiply tens? How do we divide tens? How do we multiply by tens? How do we divide by tens? In each case, the question comes as a constantly older, and more familiar, question that always calls for a common answer that gets easier and easier to sense. This is the reason why arithmetic can become less difficult as it becomes outwardly and apparently more complex.

The pupil not merely gains a growing mastery of his unifying idea, but also becomes increasingly conscious of the possibility of putting it to use. He looks into the new things set forth for him to learn, to discover where his idea may fit as means of rounding out the new things and making them sensible. In some such manner, especially if his teacher guides his attention by asking appropriate questions, he sees, as he undertakes the operations with decimals, how he may deal with tenths, hundredths, and thousandths as he has become accustomed to deal with ones and tens and powers of ten.

Other Illustrations

The pupil's work and thinking in learning arithmetic gives many other illustrations of unifying ideas which he may create and use, provided that his teacher guides his attention to the proper landmarks along the route of his learning. Following are a few such illustrations.

1. *Teens and Tens.* When the beginning pupil gives special attention to the single group of ten, both when it stands alone and when it stands with other smaller groups as the base of the teens, he gains the useful unifying idea that the teens have a common characteristic which makes them much alike. He may now learn, with this teacher's guidance, to look upon the 36 so-called "harder" additions —those having sums of 11 to 18—as the single operation of reconversion into a ten, and to attack them all with this common method of work and thinking. Similarly, he may learn to consider the 36 corresponding subtractions—those having minuends of 11 to 18—as the single operation of subtracting from a ten. In either case, the pupil gains an idea of procedure that he can use independently and confidently to determine his answers to what otherwise would appear as 36 different questions.

The pupil may extend the use of his idea of ten to the unifying of the simple multiplications which are usually a source of trouble to beginners. Thus the multiplications, "four fives" to "nine nines"— $\begin{array}{r}5\\ \underline{\times 4}\end{array}$, $\begin{array}{r}9\\ \underline{\times 9}\end{array}$—convert into the common question, "How many tens?" This common question supplies an idea of procedure the pupil can use independently and confidently to provide his answers, however much or little he learns to use other methods to supplement.

Several of my students have experimented to determine whether beginning pupils can learn and use independently methods of work and thinking, such as those just indicated, to determine answers to simple additions, subtractions, multiplications, and divisions. They have found, without regard to social background or learning speed of the pupils concerned, that all could give special attention to the idea of ten and thereby learn to use it to find their own answers independently to the questions in arithmetic put before them. Their studies merely confirmed what good teachers have long observed. Whenever the teacher makes his assignment clear, or makes his question clear, his pupils surprise him much more with suc-

cessful performance than they do with evidences of inability.

Yet there are persons concerned with the teaching of arithmetic who do not take kindly to the suggestion that pupils may learn to produce their own answers independently. They say that such methods of learning are slow and roundabout, and conform not at all to the speedy and direct responses later usage requires. These persons are they who have not learned the basic pedagogical principle that methods of learning and methods of use cannot possibly be one and the same.

2. *The Three Kinds of Problems.* When the pupil learns to multiply and divide by fractions, he soon meets the questions which we commonly classify as the "three kinds of problems," namely.

a. Finding the part of a number;
b. Finding the part one number is of another;
c. Finding the number when a specified part of it is known.

These problems confuse with their similarities, yet they have differences which the pupil can learn to distinguish. I have observed pupils in the fifth grade looking for, and frequently observing, the distinguishing features of the "three kinds of problems" in their pursuit of answers to the questions: Does the problem mention numbers only, or a part and a number? What number (in the case of the former) does the problem ask about? Is the part (in the case of the latter) of a number that *is* given, or of a number that *is not* given?

The pupil who thus learns the distinctions between the "three kinds of problems" gains a method of attack which serves to unite the three different problems somewhat as the three sides of a triangular situation. He is prepared to carry this method of attack to the later and more commonly used statements of the "three kinds of problems" as he meets them in his study of per cents. And if he continues, or has the opportunity to continue, to keep in clear consciousness the "three kinds of problems," he may find all the varied applications of percentage he must study and learn resolving themselves into the familiar three categories.

3. *Procedure in Measuring.* When the pupil looks behind the scenes of the usages of the common measures of his earlier arithmetic, he may learn to notice the two steps of thinking and practice in the determination of the sizes and amounts of things. These steps are, first, determining or choosing a suitable measure, and second, using the measure. Though the measures he uses have long since been determined for him, he may now gain an idea of the relation between a given measure and its special usefulness. The idea, of course, is no substitute for the practice the pupil should have in the uses of the measure, and it is not a needed item for the practice; yet the idea does become a valuable one when the pupil carries it to the studies of measures which to him are uncommon. He may use the idea to unify all his measuring into a somewhat similar procedure, and thus to develop the steps of the rules of measuring that have a practical use.

4. *Asking Questions of Problems.* The pupil who learns to attack the simple one-step problems of his beginning arithmetic by raising and answering the questions, 1. "What does the problem tell?" 2. "What does the problem ask?," gains thereby the means of attacking problems of more than one step as he comes to them. He has no trouble, in the case of the latter more complex problems, in moving along to the succeeding essential question, "What else must I know to find the answer?" He may not always arrive at the desired answer, but he is on his way.

The Teacher as Guide

We do not need through further illustration to labor our discussion of the pupil at work unifying his ideas in arithmetic. In every illustration we have seen the pupil as the active agent in determining what and how he learns. Yet every illustration has suggested the guiding hand of a teacher who manages the pupil's learning. Though it must be the pupil who works out the unifying that he learns and uses,

it nevertheless is his teacher who provides the sequences of his work that make it possible for him to bring it all into a pattern of unity. For example, in a certain classroom, the pupils one day were responding as follows:

Said one: "I can find seven sevens; I know four sevens are twenty-eight and three sevens are twenty-one. I add twenty-eight and twenty-one; so seven sevens are forty-nine."

Said another: "I can find nine sevens; I know three sevens are twenty-one, and nine sevens are three times as much. So I multiply, and I know that nine sevens are sixty-three."

With their oral responses, the pupils wrote:

$$\begin{array}{r}7\\ \times 7\\ \hline \\ \end{array}, \quad \begin{array}{r}7\\ \times 4\\ \hline 28\end{array}, \quad \begin{array}{r}7\\ \times 3\\ \hline 21\end{array}, \quad \begin{array}{r}28\\ 21\\ \hline 49\end{array}, \quad \begin{array}{r}7\\ \times 7\\ \hline 49\end{array};$$

$$\begin{array}{r}7\\ \times 9\\ \hline \\ \end{array}, \quad \begin{array}{r}7\\ \times 3\\ \hline 21\end{array}, \quad \begin{array}{r}21\\ \times 3\\ \hline 63\end{array}, \quad \begin{array}{r}7\\ \times 9\\ \hline 63\end{array}$$

It is merely rhetorical to ask who was the person in charge of this classroom, and who made it possible for the pupils to unify otherwise separate and diverse multiplications into a single idea of procedure.

In the proper conduct of the pupil's learning in arithmetic, the teacher need not worry about how best to provide meanings and get them unified. All the teacher needs to do is ask the right questions at the right time. The pupil creates his meanings, and does his own unifying.

EDITOR'S NOTE: Professor Wheat emphasizes pupil's thinking with numbers and the relationships of groups as represented by numbers. He says, "The paraphernalia of present-day primary arithmetic is out of all proportion to any possible arithmetical usage pupils can make of it." Does some one wish to take issue with Mr. Wheat? What is the proper role of "concrete materials"?

BOOK REVIEW

Guiding Arithmetic Learning, John R. Clark and Laura K. Eads, World Book Company, 1954. 304 pages, $3.50.

Few members of the National Council of Teachers of Mathematics need an introduction to the authors of this volume. Those who do might well consult Dr. Clark's article on the "Use of Crutches in Teaching Arithmetic" and Dr. Eads's report "Arithmetic on the March" both of which appeared in THE ARITHMETIC TEACHER for October 1954.

Guiding Arithmetic Experiences presents material that will be helpful to the experienced teacher, yet the presentation is well spaced to suit the beginner. A particularly valuable feature of the book is the constant use of statements made by slow children and by bright ones. These quotations illustrate important differences between the two groups. They also emphasize the need of analyzing the child's thinking, of seeking his point of view, of noticing the way in which his ideas sometimes outstrip his vocabulary.

The authors stress the desirability of making the transition from concrete representation to the abstract at as early a date as possible. The first chapter "From Things to Concepts" is particularly stimulating. So also is the final chapter "Planning for Effective Teaching and Learning in Arithmetic."

The authors have kept closely to the topic of *guiding learning in arithmetic.* Their concern is that arithmetic must have meaning to the child. They tacitly assume that it has meaning to the teacher. The college student using this book should be cautioned that he will need to look elsewhere to find the meaning of certain terms that are used without explanation. Examples of these are addend, commutative principle, algorism.

The more experienced teacher will note that while the authors are conscious of individual differences and varying rates of learning, they lay considerable stress on "ways in which a teacher can teach the whole class the same topic and yet provide for group teaching." But this important idea does not receive as detailed a development as one would desire. Specific illustrations such as were given in earlier chapters would have helped a classroom teacher in the difficult task of teaching the whole sixth grade the same topic when certain members of the group were retarded in arithmetic by perhaps a couple of years.

VERA SANFORD

Concrete Materials for Teaching Percentage

Eldon Hauck
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Many problems and difficulties in elementary school arithmetic can be resolved through: (1) better instructional methods, (2) more instructional materials, (3) better instructional materials, (4) concrete materials for each basic phase of the subject, and most important (5) the *know how* in using materials in the classroom. Concrete materials for teaching and learning should have: (1) the property of being handled in such a way as to conduct a learning situation to the handler or observer, (2) the inherent structure to present to the learner a sequential pattern leading to the abstract, and (3) a specific relation and application to the subject being treated which, at the same time, gives an immediate experience into reality. Concrete materials will enable the child to participate in discovery and develop organization and insight.

Leading into Percentage

Understanding of problems and ability to solve problems using common and decimal fractions provide the pupil with a background which will facilitate his understanding of percentage. A thorough review and reteaching of these fundamentals is desirable prior to introducing percentage. Stories about the invention of per cent and its use will arouse interest. Newspaper advertisements showing per cents may be brought to school and displayed and discussed, a store or bank may be visited, and pupils may talk with their parents and others about the use of percentage. These activities should pave the way to a readiness for a study of percentage. All pupils who bring things to the classroom should have an opportunity to present and discuss them with the class.

Our goal is an understanding of per cent together with an ability to solve problems. The factors which lead to an understanding of the thought processes of solutions are:

(1) Changing fractions to decimals and per cent.
(2) Changing decimals to fractions and per cent.
(3) Changing per cent to decimals and fractions.
(4) Per cent means "hundredths."
(5) Of means times (×).
(6) Per cent is a simpler way of working with fractions.
(7) To find a per cent of something is the same as finding a part.

There are two chief difficulties encountered in the teaching of per cent. These are: (1) *the inability to understand the relationship of per cent to fractions and decimals* and (2) that 100% means the whole of *anything.* A third difficulty is encountered in that, if one whole thing equals 100% then two whole things (like objects) equal 200%, etc.

An understanding of the above listed factors and a doing away with the difficulties encountered may be achieved wholly or in part with the use of the "Percentage Box" as illustrated on page ten.

Figure 1 shows the entire box as being equal to 1 or 100% or 1.00; Figure 4 (section A) is equal to $\frac{1}{2}$ or 50% or .5 and figure 2 (section B) is equal to $\frac{1}{4}$ or 25% or .25.

A combination of A and B equals $\frac{3}{4}$ or 75% or .75.

An introductory procedure of explaining the box and showing the above; with discussion and questions would show a definite relationship between fractions, per cent and decimals.

Remove section C from the percentage box (hide sections A and B) and point out that you have something in your hand, section C (figure 3) and that you hold all of this something. Ask what per cent it equals (it is important to keep the inside section printing hidden from the class). In most cases they will answer, 25%. Repeat that you have the whole thing and nothing less. They will soon deduce that you have 100%. Then show them the printing you

THE PERCENTAGE BOX

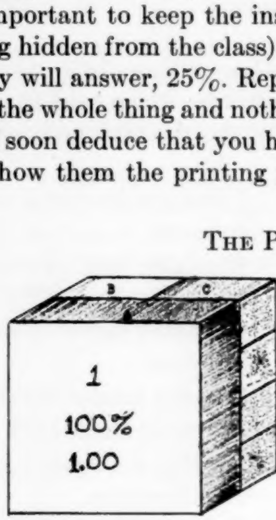

FIG. 1

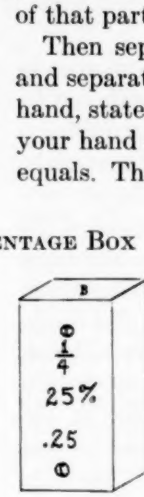

FIG. 2

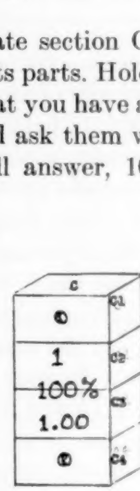

FIG. 3

The box in entirety is a cube. Its dimensions are flexible. It is held together by dowels in section A fitting into the holes of sections B and C.

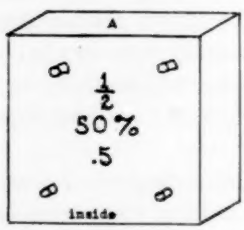

FIG. 4

Section C is held together in the same manner: C1 having the dowel and C4 having the hole with C2 and C3 each having a dowel on one side and a hole on the opposite side.

have been hiding from them. Let them decide what each part and combinations of two or more parts equal in per cent, decimals or fractions. e.g., C2 equals what per cent? What fraction, what decimal? C1 and C2 equal what per cent? What decimal?, what fraction?, etc.

Now bring the other sections, A and B, out of hiding; place all together again and go over the procedure. Point out to them (their questions will call for answers) that all of anything is equal to 100%, but a part of anything can also be equal to 100% of that part.

Then separate section C from the box and separate its parts. Holding C1 in your hand, state that you have a whole thing in your hand and ask them what per cent it equals. They'll answer, 100%. Place C2 beside C1 and say, "If C1 equals 100%, what do both of these, C1 and C2, equal?" If they do not come up with the correct 200%, repeat the process. They will! Then add C3 and C4 to the group for an eventual 400%. Work back through the processes and let the students demonstrate and ask questions about the percentage box.

The second and third difficulties mentioned above have now been surmounted. Repetition of the procedure will help the great majority of pupil's through these obstacles.

To change fractions to decimals and per cent show the whole box and ask, "How many hundredths in one whole thing?" "To find how many hundredths in one-half a whole thing, what would you do?" "If you wanted to find how many hundredths are in one-fourth of a whole thing what would you do?" "Per cent is just another name for hundredths. If you know how many hundredths are in something, can you tell what per cent it equals?" etc.

To change decimals to fractions and per cent, "You know how many hundredths are in a whole thing, how would you find what part of a whole thing fifty-hundredths equal?" "Five-tenths?" "How many hundredths do five-tenths equal?" "How would you find what part of a whole thing twenty-five hundredths equals?" "Fifty per cent?" "Twenty-five per cent?" etc.

To change per cent to decimals and fractions, "What is the difference between 100% and 1.00, 25% and .25, 50% and .5?" "What would you have to do to the per cent to make it like the decimals?" "You know how to change per cent to decimals, can you find the fractions they equal?" "These are called equivalents."

The above procedures have brought about the fact that per cent is just another way of saying hundredths (accompanied by understanding).

Bring forth the percentage box once more. "Let us assume that this box is equal to one hundred cows." "If the whole thing is equal to one hundred cows, how many cows would one-half of it equal? 50%? 25%? $\frac{1}{4}$? 75%? $\frac{3}{4}$?" etc.

To find a per cent of a number raise these questions: "Fifty per cent of one hundred cows equals how many cows?" "25% of 100 cows equals how many cows? One hunded per cent of 100 cows equals? Two hundred per cent of 100 cows equals?" etc.

Now show them the process of working the problem out on paper by changing the per cent to hundredths (decimal) or a common fraction and multiplying. And later when the above has become known to the majority, give them the following step:

To find what per cent one number is of another—"If the whole box equals 100 cows and one-half the box equals fifty cows, how would you find what per cent fifty cows is of one hundred cows?" "Are fifty cows a part (fraction) of 100 cows?" "What part?" "This part equals what per cent?"

"Twenty-five cows equals what per cent of 100 cows?," etc., etc. If 100 cows equal one hundred per cent, what per cent would 200 cows equal?" "Two hundred cows are equal to what per cent of 100 cows?" etc.

Now show the problem solving processes: "find the part, change this fraction to a decimal, change the decimal to a per cent." And later:

To find a number when its per cent of the whole is known:

"The whole box is equal to one hundred cows. Twenty-five cows are 25% of how many cows? Seventy-five cows are 75% of how many cows?" etc. "Now let's forget about the box and the hundred cows! Ten cows are twenty-five per cent of how many cows? Twenty-five cows are fifty per cent of how many? Fifty cows are 100% of how many cows?" etc.

The cows need not be used entirely, but the cow is a good example because a cow, when divided, is no longer a cow but a beef.

And now the problem working process can be brought in: finding the answer by either first determining 1 per cent and then 100 per cent, or by dividing the stated number by its per cent expressed as a decimal.

The above exercises should allow each student the knowledge that he can work the problems in his head and therefore most certainly can do them on paper. The practice involved will now be more of a part of his makeup than if the methods had been taught to him by *rote!* His understanding of all fractions should be more complete and his work habits in percentage should be of the desirable nature.

Money may also be used to illustrate further the above points. A silver dollar, 50 cents, 25 cents, dime, nickel, and a penny. The only drawback is the inability to divide the silver dollar. In its favor rests the children's familiarity with money. Once they get the idea from the percentage box, money can be used to good advantage. Whenever a difficulty arises, refer to the percentage box again and again.

Conclusion

Much has been said about and many materials supplied for learning certain parts of arithmetic such as the number system, fractions, and measures but percentages has been neglected. There is need for an interchange of ideas on teaching methods that teachers have found beneficial. These ideas should properly come from teachers experienced in working with children. Better teaching will help the individual child growing up with a feeling of courage, accomplishment, and understanding. Lack of understanding is the basis of fear and the average adult today has a fear of mathematics brought about chiefly through school experiences in this subject. The average adult holds the subject as one of extreme importance in getting along in life and berates his child, hoping the child will learn better than the parent.

Good learning of arithmetic depends upon good guidance and teaching. A good teacher needs not only how to deal with children but also he needs a good background in the materials he is teaching. In many cases this calls for in-service education of teachers in the selection and use of instructional methods and materials. Some new policies in requirements in the teacher training colleges are also desired.

EDITOR'S NOTE: Mr. Hauck has a device which he uses to illustrate concepts and principles of percentage. The device alone is probably of little significance, the importance of this or any device lies in the way it is used. The artist teacher knows when and how to use a manual-visual aid to enhance understanding and when it should be avoided or dropped in favor of mental processes alone. What do readers think of visual-manipulative materials?

THE EDITOR REQUESTS

THE ARITHMETIC TEACHER would like to publish the good teaching procedures that teachers have found particularly helpful in various circumstances. How do you handle the slow learners in grades 2, 3, 4, 5, 6? How do you provide for the very bright pupils who seem to learn in one-fourth the time required by the average? What are the "hard spots" in your grade and how do you master them? What do you do about children who can't do simple column addition in grade 4? When your children persist in counting (on fingers, with dots, etc.) to do their addition and subtraction, what is your remedy? What do you do with the youngsters who do well with abstract figuring but don't seem able to read a simple problem?

It is the aim of THE ARITHMETIC TEACHER to be of service to the schools. In future issues it is planned that more of the articles will deal with specific problems such as those suggested above. But to do this the editors need advice and materials. These materials are available in many schools. Yes, teachers are busy. But please, some teachers should spend just a few moments writing down some of their excellent procedures so they may be shared with others. Do not hesitate because you may feel your composition is not the best. It is an editor's job to help in writing. Ideas are always more precious than the formal presentation thereof. Please help the editors.

In the April issue will appear a listing of special summer workshops that may be of interest to teachers of arithmetic. To be included, announcements must reach the editor before February first.

ST. LOUIS—DECEMBER 27-29, 1954

With the convention theme "Forward with Mathematics" the National Council of Teachers of Mathematics will hold its fifteenth Christmas meeting at the Chase Hotel in St. Louis, December 27–29. Meetings devoted to arithmetic range from such topics as "Building Concepts in Arithmetic" to "An Arithmetic Laboratory" and a special "Arithmetic Conference" at which several interrogators will direct questions for discussion to special consultants.

Toward Perfect Scores in Arithmetic Fundamentals

A Study in Research

Guy M. Wilson

Emeritus Professor of Education, Boston University

During the period 1928–42, a number of graduate students of Boston University School of Education were checking on the corrective load in the fundamentals of arithmetic in the upper grades and high school, and the possibilities of better scores, possibly perfect scores, in the fundamentals. Several of these students produced masters' theses which are now on file in the library of the School of Education.[1] Some of the details of these studies which are of interest to teachers in junior or senior high schools or colleges are reported herewith.

The first sizeable study undertaken was by Sweeney (1929) who worked for perfect scores in long division, with 70 fifth grade pupils and 38 sixth grade pupils; of these, 62 fifth grade pupils and 36 sixth grade pupils reached perfect scores before the end of the year. Since long division involves multiplication, and subtraction, it was necessary to bring pupils to perfect scores in these processes, and that required half the year.

Some of the most convincing evidence of the desirability and feasibility of the 100% program for the fundamentals resulted from the work of Yeomans and Parsons at the Paul Revere School in Revere, Mass., and the work of Sweeney at the Gridley Bryant School in Quincy, Mass. In these schools the 100% results were secured and maintained for every normal pupil,—by deferring drill to the third grade for addition, and other processes in grades above, by mastering one process at a time, by regular checking and re-teaching as needed, by use of a scientifically constructed drill service, and by proper motivation and direction of pupil efforts.

Data on results in addition only will be presented in this article. Addition results are typical; subtraction results are about the same; multiplication and long division results are not as good, but similar. In most schools the scores are too low to justify any quibbling over them. Class averages of 75 or 85 can be attained without a single pupil making a perfect score, and many of the pupils may be counting instead of adding.

Accuracy as a goal in arithmetic is no doubt accepted by all. The drive for accuracy can be limited to processes used enough in life to justify mastery. There is adequate research to support the statement that over 75% of adult figuring is in the fundamental processes,—addition, subtraction, multiplication, and division.

Specialists such as accountants and engineers use more of the processes beyond the fundamentals, such as decimals, percentage, and proportion. But always the basic figuring is in the fundamentals. College and university professors who teach higher mathematics report that a large proportion of the mistakes made by their students are in simple arithmetic. The goal of accuracy in the fundamentals of arithmetic is, therefore, acceptable and imperative to all groups and at all levels.

Accuracy in the fundamentals would

[1] See Masters' theses, Boston University School of Education: Sweeney, 1929; Hammond, 1929; Burgess, 1930; Miller, 1930; Varney, 1932; Soles,* 1935; Caton,* 1936; Randall,* 1936; Ridlon, 1936; Hanley, 1938; Yarbrough,* 1938; Houghton,* 1939; Gilmore,* 1939; Buckley,* 1940; Earle,* 1940; Rockwood,* 1940; Ringer,* 1940; Bancroft,* 1941; Lund, 1941; Drew,* 1942; Gray,* 1942. In many cases inter-library loans of these theses can be arranged. Theses of those whose names are starred are on file with the Office of Education in Washington, D. C.

appear to be a feasible goal. The load is not heavy,—in addition 100 primary facts plus 300 decade facts to 39+9, plus 80 decade facts for carrying in multiplication, —a total of 480 addition facts. In subtraction, there are 100 facts, no more. In multiplication there are 100 facts. In short division (now a discontinued process, in most schools) there are 81 even division facts helpful in long division. In long division, well developed process steps and a scientific plan for teaching are needed. There are process steps helpful in teaching addition, subtraction, and multiplication also, and in each process there should be a simple development plan which a pupil in the grades can understand in initial learning, and which can be used in high school or college for re-learning or remedial work.

It is inexcusable, but true, that many teachers do not know how many addition facts there are, and have no organization of these facts for teaching. They merely assign pages in a text, and the average text has no adequate organization for mastery.

A total of 21 masters theses were completed at Boston University dealing with deficiencies in the fundamental of arithmetic. In some cases the survey of needs was followed by teaching for the removal of deficiencies. Due to travel, time limitations, and inability to direct the time to be spent by the pupils except with the teachers' cooperation, it was not possible in every study to bring the chosen group to perfect scores, although in every case marked progress was made, and usually the regular teacher became interested.

Discovery of Deficiencies

Randall, 1936, tested 200 pupils; 100 in the seventh grade and 100 in the eighth grade. His sample included the first 100 in each grade according to the alphabetical arrangement of names. Of the 200 pupils tested, 37 or 18.5% made perfect scores in addition in this preliminary testing.

Nelson, 1938, tested 1215 junior high school pupils, 450 in grade seven, 400 in grade eight, and 365 in grade nine. The percentages of perfect scores in addition were: seventh grade, 12.47%; eighth grade, 12.50%; and ninth grade, 10.00%. Over 1000 of the 1215 pupils needed corrective work.

Gilmore, 1939, tested 174 pupils in grades seven to twelve. The percentages of perfect scores in addition for these grades were, respectively, 17%, 20%, 13.22%, 15%, 7.4%, and 32%. (Only the best twelfth graders remained in school.)

Buckley, 1940, working in a large senior high school with the first fifty (according to the alphabet) pupils in each of the three grades, found perfect scores in the four different courses offered, as follows:

	College	*Industrial Arts*	*Citizenship*	*Commercial*
10th grade	10%	6%	12%	18%
11th grade	20%	26%	12%	20%
12th grade	8%	12%	12%	30%

Ringer, 1939–40, worked with 27 junior high school pupils through two years, grades seven and eight. At the first checking one pupil, or 3.7%, made the perfect score.

Houghton, 1939, working with a small group of junior high school pupils, found 12% with perfect scores in addition.

Earle, 1940, testing 323 eighth grade pupils, found 47, or 14.21% perfect on Wilson's addition process test. Her best eighth grade of 30 pupils showed 36.63% perfect.

Kyte, 1940, summarized the results from the testing of 659 college and university students. Many of these were teachers. A total of 526 tested without previous announcement showed 42.2% with perfect scores. A group of 47 at Atlantic Union College, following a previous announcement, showed 44.7% with perfect scores. A group of 86, told in advance to prepare for the test, showed 67.5% with perfect scores.

Lund, 1941, tested 1050 students in a large senior high school in the fundamen-

tals of arithmetic. In addition, 13.51% made perfect scores. In subtraction, the perfect scores averaged 18.19%, in multiplication, 3.93%, and in long division, 5.52%. These results were based on simple tests in whole numbers only, the Wilson process tests.

Lund distributed his data by sex, by courses—college, commercial, and social arts, and by years—tenth, eleventh, and twelfth. All students in the high school were included, not just samples. In general, the boys did a little better than the girls, and the commercial group did better than other course groups. The best average score was made by the tenth grade commercial boys, with 34.48% of perfect scores in addition. The poorest showing was made by the 66 tenth grade girls in the college course; they showed only 7.58% perfect scores on the addition process test.

Each of the above studies involved much more detail than here reported. Most of them show results for all four of the fundamental processes, and they go into detail as to procedures.

Three addition tests were used, but the results above are based upon a process test, which is a simple test of 25 examples, whole numbers only, and with no combination above 39+9. The most difficult example involved adding in U. S. money, seven addends, each less than $100. There was no time limit. University students generally finished in four to six minutes; when the time exceeded eight minutes, there was usually evidence of counting.

The above reports on deficiencies in the fundamentals of arithmetic are fully confirmed by a W.P.A. study[2] of 17,700 pupils in grades six, seven, and eight in the fall of 1935 in towns and cities in the Boston area. Perfect scores in addition in fourteen of these cities varied in the sixth grade from 8% to 19%; in the seventh grade from 6% to 18%; and in the eighth grade from 8% to 19%: in the fifteenth city perfect scores in addition ran to 89% and 90%. The children and teachers in this fifteenth city were in a school under the principalship of one of the students who had completed a study on perfect scores in the fundamentals.

Average (median) scores and low scores are also interesting. Median scores in the fourteen cities referred to above, in the seventh grade, for example, were either 88 or 92; in the one city the median score was 100. Low scores in the fourteen cities ranged from 36 to 60; in the *one* city, the low score was 96. A score of 96 means one wrong answer on the 25-example test.

Corrective Work

Most of the 21 studies ended with a survey of deficiencies in the fundamentals. Others undertook some corrective work, and a few of these were with junior and senior high school pupils.

Randall selected seven junior high school pupils, each of whom had: (1) low scores in arithmetic, (2) high scores on intelligence, (3) a willingness to cooperate in the experiment, and (4) were available at a convenient time. These seven all attained the 100% score.

Gilmore selected four cases with low scores in arithmetic and intelligence of average or higher. In from six to ten periods he brought them to perfect scores.

Ringer worked for two years with 27 junior high school pupils. The seventh graders were advanced from 3.7% perfect scores to 59.2% perfect scores; the time spent on corrective work was one hour per week for six months. A similar schedule of corrective work with these same pupils as eighth graders advanced the percentage of perfect scores to 88.8%.

Houghton, working with seventh and eighth graders applied corrective procedures in all four fundamental processes.

[2] *The Corrective Load in the Fundamentals of Arithmetic in Grades Six, Seven, and Eight in Fifteen Representative Towns and Cities of the Metropolitan Boston Area.* W.P.A. Project No. 17-C, United States Office of Education, under the direction of Guy M. Wilson, 1936.

There was a 52% gain in the number of pupils reaching perfect scores in the four processes. The time was one period per week for one semester.

Earle did corrective work with the 323 eighth grade pupils in her study. She tried to influence the teachers to do something about the 85.79% of their pupils needing help. She remarks: "Many of the eighth grade teachers feel no obligation to do any corrective work in arithmetic fundamentals." However, perfect scores on the addition test moved from 14.21% in October to 29.05% in March for eighth grade pupils.

Nelson, in her work with 1215 junior high school pupils, noted that "carrying" was a cause of many errors.

Significant Comments

Here are the comments of a few of the students cooperating in the program for perfect scores in the fundamentals.

"Poor teaching is a greater handicap than dull children." (Sweeney, Mastery of Long Division at the Fifth Grade Level.)

"The teacher wastes his time when he presents more and more complicated arithmetic situations to the already bewildered student." (Buckley, Senior High School Investigation.)

"The habit of checking is not common, but when acquired, is a great aid to accuracy. Checking should become automatic, as a part of the solution." (Gray, Study in the Junior High School, Grades Seven and Eight.)

"Teachers in many schools are so bound to schedule that they fear to spend time on work that has been scheduled for a lower grade." (Yarbrough, The Arithmetic Needs of 127 Pupils in the Sixth Grade.) Miss Yarbrough in the first testing of 127 pupils, found only six with perfect scores on the addition process test. This school system was ranked by itself and others as one of the best in the country.

"Indifference of secondary school teachers to deficiencies in the fundamentals of arithmetic must change; the secondary school must accept the problem as its own." (Lund, Senior High School Study.)

"A wholesome attitude toward arithmetic is promoted when the child feels (1) that arithmetic is useful, and (2) that success for him is possible." (*A Curriculum Guide for Intermediate Grade Teachers*, Commonwealth of Mass., Dept. of Education, 1951, pp. 39–40.)

"These figures are most significant and indicate that 100% results are possible in the fundamentals of arithmetic." (Gilmore, Junior-Senior High School Study.)

"No corrective work will be effective unless it is properly motivated." (Nelson, Junior High School Study.)

Conclusion

In this article an effort has been made to present data for study and conclusions. It is evident that traditional procedures provide the child with a maximum of discouragement and defeat. Good teaching and regular follow-up can change this. Maintenance of drill results requires regular review and practice; this is true whether it be the piano, the typewriter, or addition. In arithmetic drill is begun too early and extended to little-used processes, and this means defeat for the essential drill program.

Mental discipline as an excuse for mastery of tasks little-used and poorly understood has been studied experimentally, and found wanting. The child does better thinking, concentrates better and develops better attitudes when he sees the value of what he is doing. Social values carry the essential meanings for the child for work in the fundamentals; mathematical meanings can be left to the teacher. Mastery of the fundamental processes in arithmetic will reduce failures in mathematics in junior and senior high school and in college.

A change in attitude is often a necessary first step in a remedial program for arithmetic at high school and college levels. Advanced students improve rapidly, with a little judicious help and guidance.

EDITOR'S NOTE: Readers of Dr. Wilson's article in manuscript raised a number of interesting questions about the content of the arithmetic curriculum and methods of teaching. Is the content to be limited to excellence in comparatively limited computations with little or no emphasis upon understanding of arithmetical concepts, ideas, and principles and with no attention to thinking and forming judgments? Is Mr. Wilson's apparent view of the curriculum as inferred from this article anywhere near adequate? Is he committed to a program of memory through drill? Are results in addition not better in other sections than in the Boston area? Do the methods employed by Dr. Wilson's students provide lasting results or do their pupils soon forget? Could these pupils who were so low in adding ability in grades seven and eight add fairly well when they were in grade four? How much and perhaps equally important arithmetic is lost to a class when 100% accuracy in certain operations is set as the goal for all pupils?

Readers of Dr. Wilson's article will be appalled by the poor results found on the tests used. He shows that better results can be attained and thus bears out the old truism "schools can achieve just about any reasonable result that they honestly try to achieve." The article points out the responsibility for each grade to refresh and reteach when necessary those things that normally belong in preceding grades. What do readers of this journal say about this article?

The Boy Who Did Not Like Arithmetic

CLARICE WITTENBERG
Laramie, Wyoming

THE EIGHT-YEAR-OLDS in the University of Wyoming Elementary School were making plans on the first day of the fall term for the year ahead. When one of the group protested at the thought of studying arithmetic, the other children were dumbfounded. From all sides they began to tell him, in no uncertain terms, just what would happen to him if he knew absolutely nothing about numbers.

One of them went a step further. "Why don't we make a play," he asked, "about a little boy who didn't like arithmetic?" His idea captured the immediate interest of the others. "Of course!" said one. "We could show a lot of things that would be sure to happen to him." The protester showed such acute embarrassment that someone suggested that the leading character should be given a different name, "Tommy, for instance." Committees volunteered and met in different parts of the classroom to talk it over and suggest number situations which could be used in the script. Then the whole group came together to pool their ideas.

Here is the play as they dictated it in final form to their teacher. Full of minor flaws and abrupt changes? Perhaps, but is it not illustrative of what we mean today when we speak of creative play-making? Best of all, it indicates a realization on the part of the children as to the role of numbers in their own lives.

Twice the group presented the little drama for real audiences. The first performance occurred at the big fall planning conference at which pupils, parents and teachers came together in the third-grade classroom to plan cooperatively. On the second occasion, the class presented it in front of a high-school mathematics group in which there were several older brothers and sisters.

The Boy Who Did Not Like Arithmetic

CHARACTERS

The announcer	A lady
Tommy, the boy who did not like arithmetic	The post office clerk
His friend Jon	The bus driver
The other third graders	The candy clerk
The teacher	The policeman
The principal	The property man

Time: First day of school
Scene One: The third grade room

Teacher: What things do you boys and girls really want to do this year?
Jon: Let's paint pictures.
Jimmy: I want to play with clay.
Karen: Will we get to take trips?
Marilyn: I wish we could read lots of books.
Elaine: Maybe we can make up some stories too.
Lowell: I'd like to collect all sorts of things, like bugs.
Sarah: In the second grade, we made an experiment to see if a magnet would pick up things under water. Will we get to make experiments in the third grade?
Teacher: I think we'll have a chance in the third grade to do all of these things and some others too.
Tommy: We won't have to study arithmetic, will we?
All the others: Oh, *yes!*

Teacher: Why, Tommy, don't you like to study arithmetic?
Tommy: No, I don't.
Teacher: It's time for arithmetic now. If you don't want to study with us, Tommy, you may take your library book to the back of the room and read.
Tommy: Oh, goodie!

(The third grade has class while Tommy turns his back and reads. After class everybody except Tommy and Jon goes home.)

Scene Two: After School

Jon: Tommy, do you remember those letters we wrote in school today? I want to mail mine. Come and go with me to the post office to buy a stamp.
Tommy: All right. I want to send mine off, too. I really must ask my father though.
Jon: Why don't you go to the principal's office and call him on the telephone? He doesn't mind if it is very important.
Tommy: I guess I will. Wait for me. (He goes to the telephone but just stands and looks at it. Then he takes the receiver down.) Hello! Hello! Father? Hello! Hello!

(The principal comes in.)

Principal: Why, Tommy, you have to dial before you can call your father. What is his number?
Tommy: What do you mean—dial?
Principal: Would you like for me to show you?
Tommy: No, thank you! I guess I'll just go ahead anyway. (He goes back to Jon.)
Tommy: I can go with you, I'm sure.
Jon: Good! Come on. (They start down the street. They meet a lady.)
Lady: Will you please tell me how to find the University?
Tommy: We just came from there. You go up the street the way we came. (He points.)
Lady: How far is it?
Tommy: I don't know.
Lady: Oh, dear! I wish I knew.
Jon: We're at Third Street and Ivinson Avenue now. Go six blocks east and you will see the University.
Lady: Thank you.
Tommy: What's the big round thing on top of that church tower?
Jon: Why that's the cathedral clock.
Tommy: What's it for?
Jon: It tells time. Listen! (The clock strikes four.)
Tommy: That's funny, it didn't tell me anything. (They go on to the post office.)
Jon: May I have a three-cent stamp, please? (He gives the clerk a nickel and gets the stamp.)
Clerk: Three cents. Four cents. Five cents. (She gives him the stamp and two pennies.)
Tommy: I want one, too.
Clerk: What kind of stamp do you want?
Tommy: I don't know.
Clerk: You will have to tell me.
Tommy: Oh, I guess I don't want one after all. Let it go. (Tommy and Jon leave.)
Jon: Here's the bus. If we run, we can catch it. (He gives the bus driver a dime. Tommy tries to get on too.)
Driver: Just a minute, little boy. Where's your dime?
Tommy: What do you mean—a dime?
Driver: You can't get on without a dime. That is ten cents.
Tommy: Oh, well, I'm not ready to go home yet. (The bus drives off and Tommy looks around.)
Tommy: Oh, goodie! There's a candy store! I like candy. (He goes inside.)
Tommy: I want some candy.
Clerk: How much do you want?
Tommy: Just give me one of those. (He points to a candy bar. The clerk hands it to him.)
Clerk: Wait a minute. Give me a nickel.
Tommy: What's a nickel?
Clerk: A nickel is five cents.
Tommy: I don't know what you are talking about.
Clerk: Then give me back that candy bar. (Tommy leaves the candy shop. Then he begins to cry.)
Tommy: I'm hungry. I don't know how to get home.
Policeman: Are you lost, little boy?
Tommy: Yes, I am.
Policeman: Tell me your street address and I'll take you home.
Tommy: It's somewhere on Fremont Street.
Policeman: What is the number of your house?
Tommy: I don't know.
Policeman: Well, what is your telephone number?
Tommy: I don't know that either.
Policeman: No wonder you're lost! Then tell me your name and I'll hunt up your street number and telephone your house.
Tommy: My name is Tommy Grant. I wish I knew all these things people keep asking me. I guess I need a lot of arithmetic.

Rationalizing Division of Fractions

Sam Duker
Brooklyn College, New York

An example of a process which is easily rationalized but still taught by rote with little, if any, explanation is the process of dividing by a fraction. Not only are children merely taught to "invert the divisor and multiply" but teachers of arithmetic are also taught to carry out this process as a rote procedure without rationalization or understanding.

A great deal has been written and much research has been done in recent years to emphasize the doubtful value of extended instruction in the arithmetical manipulation of common fractions in the elementary school (or elsewhere, for that matter). Certainly there has been a great de-emphasis in this area and children are not subjected quite as frequently as they used to be to the complicated and impractical fractions of the 33/71 type. Of all manipulatory processes involving fractions, the one least frequently encountered in real life situations is the process of dividing one fraction by another. It is possible to find some instances where an integer or a fraction needs to be divided by a smaller fraction but instances of division by a larger fraction such as in: $\frac{1}{2} \div \frac{3}{4}$ practically never occur. (It should be borne in mind that if a recipe calls for $\frac{1}{2}$ of a cup, making $\frac{3}{4}$ of the recipe calls for multiplication rather than for division.)

It is not the purpose of this article to espouse the teaching of the process of dividing by common fractions in elementary school arithmetic. It *is* the purpose of this article to point out that:

1. This process *is* being taught rather universally in the elementary schools, at the present time.
2. The process is taught without explanation by teachers who usually have no understanding of the rationale underlying the short cut of "inverting the divisor and multiplying."
3. There is an extraordinarily simple rationale for the process which at a very minimum should be understood by the arithmetic teacher. This writer takes no position here on whether the explanation is either necessary or desirable for the pupils, although a purely personal opinion, not based on research, is here expressed that the process would be much more effectively remembered if its basis were explained to the pupils as well.

Division by Fractions

A rather thorough examination of current arithmetic textbooks and of current courses of study fails to reveal any instance where the subject of division by fractions is completely omitted. It is true that there is a great contrast between textbooks and courses of study and their counterparts of twenty-five years ago in the degree of complexity of the fractional processes dealt with. Nevertheless, there is still an emphasis on such processes as division by fractions with little regard to whether the divisor is larger or smaller than the dividend. There are many reports of research showing that the process of division by a fraction is not well understood, a finding which is easily verified by almost every arithmetic teacher.

In a recent investigation Orleans[1] found that out of a total of 322 teachers and college students preparing to be teachers only 19% were able to give a satisfactory answer to the following question:

> Why is it that when we invert the denominator (in dividing a quantity by a fraction) we come out with the right answer, as in this illustration:

$$\frac{4}{\frac{3}{4}} = 4\times\frac{4}{3} = \frac{16}{3} = 5\tfrac{1}{3}$$

Not only did 4/5 of those undertaking to answer this question fail to give a correct explanation, but such a variety of illogical answers was given that the result is a clear demonstration of the failure of these teachers and prospective teachers to have any valid notion of the rationale underlying the procedure. This overwhelming result would seem to indicate that the failure was probably not entirely due to the rather unfortunate wording of the question.

No arithmetic textbook that this writer has examined provides a satisfactory explanation of this short cut and only Brueckner and Grossnickle[2] of a number of books on methods of teaching arithmetic give anything approaching a satisfactory explanation of the process. The other writers either (1) advocate the acceptance of the process as a sound device which "results in the right answer" as may, of course, be established by empirical observation and use of fractional materials such as circles, or (2) give the rather difficult-to-follow explanation that this process is a short-cut method for making the units of both dividend and divisor equivalent so that division can be performed, thus:

$$4\div\frac{3}{4} = \frac{16}{4}\div\frac{3}{4} = \frac{16}{3} = 5\tfrac{1}{3}$$

or

$$\frac{3}{4}\div\frac{2}{5} = \frac{15}{20}\div\frac{8}{20} = \frac{15}{8} = 1\tfrac{7}{8}$$

To use the short cut, we write:

$$4\div\frac{3}{4} = 4\times\frac{4}{3} = \frac{16}{3} = 5\tfrac{1}{3}$$

or

$$\frac{3}{4}\div\frac{2}{5} = \frac{3}{4}\times\frac{5}{2} = \frac{15}{8} = 1\tfrac{7}{8}$$

The rationale then given is that inverting the divisor and multiplying yields the same results as the common denominator method. That the same results are obtained is incontrovertible, but the explanation is an indirect one which fails to satisfy. The correct explanation is much more straightforward and much easier to follow.

Rationale of Division

In order to understand the rationale behind the short cut of "inverting the divisor and multiplying" the following concepts must be clear in the learner's mind whether that learner be the elementary school pupil or the teacher of arithmetic at that level:

1. An understanding of the process of multiplication of fractions.

Since in all arithmetic textbooks and in all courses of study examined by this writer the subject of multiplying by common fractions precedes any discussion of division by such fractions, this seems to be an understanding that can reasonably be expected to exist. In fact, this understanding must be possessed by the learner even when he learns the process of dividing by a fraction as a rote bit of hocus-pocus.

2. An understanding of the meaning of a reciprocal and the reasoning behind the

[1] Orleans, Jacob S. "The Understanding of Arithmetic Processes and Concepts Possessed by Teachers of Arithmetic." Publication Number 12, Office of Research and Evaluation, Division of Teacher Education, College of the City of New York, 1952. Processed. Pages 25–29.

[2] Brueckner, Leo J. and Foster E. Grossnickle. *Making Arithmetic Meaningful.* Philadelphia, The John C. Winston Company, 1953. Pages 383–396.

outcome that any quantity multiplied by its reciprocal yields a product of 1.

This should be a relatively easy concept to introduce in connection with the teaching of multiplication of fractions. This is, it is true, a concept which is not always directly developed in arithmetic textbooks or in books on methods of teaching arithmetic.

3. An understanding that in any division process the quotient is not changed by multiplying both the dividend and the divisor by the same quantity.

This is a concept which certainly must be clear to anyone who has acquired a true understanding of the process of division and is one that will be extremely useful when decimals are encountered in order to realize that,

$$.7\overline{)75.6} = 7\overline{)756}$$

or that

$$.7\overline{)756} = 7\overline{)7560}$$

The explanation of the process of dividing by a common fraction rests on these three understandings. If we wish to perform the operation:

$$\frac{3}{4} \div \frac{2}{5}$$

it is obvious that the process would be simplified if the divisor could be reduced to a value of 1. This may be done by multiplying it by its reciprocal 5/2. Bearing in mind the fact that both the dividend and the divisor may be multiplied by the same quantity without affecting the value of the quotient, we may so change the divisor provided that we also multiply the dividend by the reciprocal of the divisor, thus:

$$\frac{3}{4} \div \frac{2}{5} = \left(\frac{3}{4} \times \frac{5}{2}\right) \div \left(\frac{2}{5} \times \frac{5}{2}\right)$$
$$= \left(\frac{3}{4} \times \frac{5}{2}\right) \div 1 = \frac{3}{4} \times \frac{5}{2}$$
$$= \frac{15}{8} = 1\tfrac{7}{8}$$

or, to generalize:

$$\frac{a}{b} \div \frac{c}{d} = \left(\frac{a}{b} \times \frac{d}{c}\right) \div \left(\frac{c}{d} \times \frac{d}{c}\right)$$
$$= \left(\frac{a}{b} \times \frac{d}{c}\right) \div 1 = \frac{ad}{bc}$$

It is obvious that because the generalization involves very elementary algebraic concepts it may be unsuitable for elementary school pupils. Its value for the arithmetic teacher is, however, apparent.

Finally the intermediate step is omitted and the work appears as:

$$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8} = 1\tfrac{7}{8}$$

or, in generalization as:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

A meaningful and understandable rule for the learner may be stated: "*in order to divide by a fraction we multiply the dividend by the reciprocal of the divisor.*"

Summary

1. Despite the fact that the value of teaching arithmetical computation with common fractions is in doubt, this sort of computation is taught in most elementary schools.

2. Modern theory of teaching arithmetic suggests that short cuts are taught most effectively when the rationale underlying such processes is thoroughly understood by the learner.

3. While some arithmetic processes cannot be thus explained, the short cut for division by a fraction can readily be rationalized and understood even by the unsophisticated learner. Notwithstanding, the rationale behind this process is rarely understood either by the elementary school pupil or by his teacher.

4. Basic to the understanding of the rationale behind the rule that "in order to divide by a fraction we invert the divisor and multiply" is an understanding of: a.

multiplication of fractions; b. the concept of a reciprocal; and, c. the fact that in the process of division the quotient remains the same when both dividend and divisor are multiplied by the same quantity.

5. The rule "invert and multiply" is best explained as being a short cut for the process by which both the dividend and the divisor are multiplied by the reciprocal of the divisor, thus making the divisor equal to 1.

EDITOR'S NOTE: Division of fractions and particularly division by a fraction has always been difficult to explain on a rational basis which pupils can understand. Mr. Duker has applied the algebraic explanation to the process with numbers. What do readers of THE ARITHMETIC TEACHER think about this procedure? Who has a better method?

BOOK REVIEWS

Teaching Arithmetic in Grades I and II, George E. Hollister and Agnes G. Gunderson, D. C. Heath and Company, 1954. 168 pages, $2.50.

This book sets out to do two things: (a) to give the professional information needed as a background for teaching arithmetic in the primary grades and (b) to show in detail how to guide children in their learning experiences in grades one and two. The book is carefully planned and thoughtfully written so that both beginning and experienced teachers will find real help in it. It is well illustrated and this is important because so much of the arithmetic of young children comes from thinking about the number relationships of the things with which they deal in their daily activities.

Some teachers will not agree with the goals suggested for achievement by the end of grade two. However, the modes of learning suggested are such that any teacher can carry the work further if desired. In general, the devices suggested are both interesting and mathematically meaningful. This reviewer is a little puzzled as to why the authors include an illustration of a "five and two bead" abacus because he has found that the poorer type to use with young children.

The real test of a professional book such as this is in its usefulness to teachers. Teachers in the reviewer's class this past summer called this "an excellent book." Certain sections such as "Number Concepts of the Pre-School Child" and "Developing the Vocabulary Needed for Number Work" were found helpful by kindergarten teachers.

BEN A. SUELTZ

The Teaching of Arithmetic, second edition, Herbert F. Spitzer, Houghton Mifflin Company, 1954. 416 pages, $3.50.

This second edition offers a significant revision of the original text. There are deletions as well as additions of material. Several chapters have been cut, others have been substantially altered or expanded, a new chapter on instructional equipment, including the use of textbooks, has been added. The result is a better balance of emphasis and a more practical guide to the teaching of arithmetic.

As was the case in the first edition, the author concerns himself mostly with teaching methods while alluding to the theory of arithmetic in a casual way only. The student should, therefore, combine this text with a standard reference book on theoretical arithmetic. Teaching procedures are presented in great detail and with clarity, an especially useful feature being the large number of illustrative examples and sample lessons. The author recommends "the developmental method" of teaching, which relies heavily on the discovery of processes and reasons by the learner and presumably ensures understanding.

The matter of understanding "the meanings of arithmetic" is not stressed nearly as much as in the first edition or in many current publications on the subject. Evidently the author has arrived at the conclusion that there can be too much of a good thing, and he realizes that some pupils will have to use arithmetic in adulthood even if they do not fully understand what they are doing. On this point, as well as on some others, the book is distinguished by a commonsense approach that shows the mellowing effect of experience. The enjoyment of the book is increased by a welcome absence of verbosity.

Nevertheless, one may disagree with some statements, such as the ones dealing with the insertion of "and" between the number names of an integer, and one may regretfully note certain omissions, of which the neglect of the ratio concept in the upper grades and the lack of any reference to evaluative judgments concerning the relations between quantities seem the most conspicuous to this reviewer.

PAUL R. NEUREITER

PUZZLE

A boy scout troop had nine pieces of chain which ranged in length successively 1, 2, 3, . . . 8, 9 feet. How can they combine these pieces so that they will then have three pieces of equal length?

Peter Is a Slow Learner

Lois Vincent
Oneonta, New York

Peter is eight years and nine months old. His tests say he is six years and one month old. He has been in school three years, and is now ready for the arithmetic taught in the second half of the first grade in his school.

Peter is one of sixteen pupils in the remedial grade, each needing special attention in one or more areas. The age span of the group is from seven to fifteen years. Everyone in the group is from one to five years behind his reading level. Each child reads individually every day, and phonics work is stressed greatly. Arithmetic processes are taught, not once or twice, but dozens of times. Group Social Studies are taught. The older boys have shop work daily, and everyone has work in many handcrafts.

For two years Peter has seemed the most hopeless case in the group. Peter will be a citizen who is likely to remain in the background of the community where he was born and raised. Peter's training in school needs to bring to him the knowledge necessary to make the best possible citizen of him: in arithmetic, the four fundamental processes, measuring, a knowledge of money and banking, a knowledge of comparison of values, using time to the best advantage. As Peter will probably leave school at the age of sixteen, this is an ambitious program.

Peter spent one year in the regular first grade, and for his second and third years in school, has been in a remedial grade.

When Peter entered the remedial grade he could only color pictures. Now he can count by 1's to 500; count by 5's and 10's to 100; write his numbers to 100, and add two numbers as long as neither is over ten.

First Peter learned to count to five. He counted the teacher's fingers for days, until he could do the job correctly every time. Then using both of the teacher's hands he learned to count to ten. Peter counted books, he counted the number of people in each reading group every day, and counted out workbooks to each child in the group. He counted paper and pencils and everything else the teacher could find for him to count.

The numbers 1 to 10 were written on sheets of paper and on the blackboard, and Peter copied them. Drill, but we had to start somewhere.

While this process was being carried out, the teacher found a "connect-the-dots-book," with numbers in the first half that only went up to ten. The numbers 1 to 10 were written on a strip of oak tag and Peter started to use the book. He would find 1 on the page of the book and put his pencil point on it. He put the first finger of his left hand on the 1 on the oak tag strip and he was ready to go. He would move the finger down on the strip to the 2, study it, and then find the same number in the book. He then drew the line from 1 to 2. Then he moved the finger on the oak tag down to 3, and the process was repeated. The fours had to be made as 4 because they were that way in the book, and 4 and 4 did not, and still do not look alike. Some day he may recognize both. This process went on until Peter reached page 56. This was the first page completed without his crutch, the oak tag strip. Later the dot book was laid aside for the next year as the rest of the pages had dots to 25.

One day it was discovered that Peter had ten toes, so he counted his toes as

well as his fingers. In fact, he said to the teacher, "I've got ten at the top and ten at the bottom."

Peter was given his tests in June with the first grade mental age five years three months.

The second year that Peter was in the remedial grade, two of his brothers were in the same group. David, the older, a serious, responsible boy of fourteen—mental age eleven years six months—tried to help Peter all year.

Peter remembered some of the arithmetic he learned the year before, he forgot some too. A short review prepared him for further explorations. Another oak tag strip with numbers 1 to 25 was prepared, and the "connect-the-dots-book" was put in use again. It took Peter only ten days to master the fifteen new numbers, and in twenty-three days he had finished the remaining forty-six pages in the book.

About this time Peter started counting to 25 aloud. Sheets of paper, pencils, books, and pressedboard discs were counted. These discs were Peter's pride and joy for some time. Twenty-five were counted and piled high; they were counted and set on their sides in a small box into which the twenty-five just fit.

His brother, David, spent quite a bit of time at home working with Peter, and Peter came in to the teacher one morning and said he could count to one hundred, and he did it. "Now can I write to 100?" was his question. A stencil was prepared, with one-hundred double boxes. There were ten rows of ten boxes. The teacher wrote the numbers in one row and Peter copied them in the other beside it. By the end of the school year, and about a ream of paper later, Peter could write his numbers to 100 on a one-hundred single-box sheet. Peter counted the numbers as he wrote them, and every one of the numbers to one hundred could be recognized out of context by the end of the year.

At the beginning of the second year, the teacher also produced a set of number blocks. These were gaily painted blocks of nine different heights. Numbers were painted on the blocks. The "9" block was the largest, and the "1" block the smallest. Nine "1" blocks would be the same height as the "9" block. For some time Peter just played with the blocks, but after a bit, the teacher saw him working with pencil and paper when he had the blocks at his desk. On investigation it was found that he was copying down the numbers on the small blocks and the one larger one in a series 3, 2, 2, 1, 1, 9. At this point Peter was introduced to addition.

Discs in the same number as the blocks were put beside them and counted. It was discovered that the numbers always came out the same. The mechanics of setting down an addition fact on paper was explained and practiced for several days. The addition sign was necessary, and Peter was off. He filled papers with combinations he made, and soon outgrew the discs and blocks.

The teacher noticed, at this time, that when Peter made a $\underline{6}$ or a $\underline{9}$ there always was a line under it. Investigation showed that both these numbers on the blocks had this identification on them. Nothing was said about this to Peter. Maybe it will persist and maybe not, no matter either way.

In conjunction with this addition discovery, flash cards with combinations of two numbers up to ten were introduced. Peter drilled on these with the teacher, with other pupils and alone.

Discoveries were made about adding one and zero to numbers, and were correctly learned in combinations up to one hundred.

At this time drill sheets were introduced, and Peter felt he was accomplishing something. He was doing drill sheets like everyone else in the room.

A simple two-wire forty-bead counter was purchased, and Peter did not need any help to put this to work. He worked it out for himself in two days. However, he soon lost interest in this device.

Toward the end of the year, the teacher

discovered that Peter had no idea about money. The teacher sent ten dollars to the bank, and got ten dollars in change. This was the money the teacher intended for her petty cash fund.

This was a new approach to the other children in the class. There was any amount of help to teach Peter the names of the coins, and the teacher tried to teach him the value of the money. Peter counted the coins a dozen times a day to see it was all there. The money was kept in a small plastic box, and Peter was custodian of the box. Woe be unto anyone who touched that box.

Peter needed a pencil, so he was taken to the store to buy one. He had the little plastic box with the ten dollars in it. He asked the storekeeper how much he had to pay for the pencil, and very reluctantly parted with a nickel. At this point he refused to buy anything more, even though he needed paper too.

The teacher contacted the owner of the hardware store, and when anything had to be purchased there, Peter went along and had to pay for the purchase with some of the money from the plastic box. Sometimes it took quite a few minutes for Peter to count out the right money, but everyone was patient and Peter learned by doing.

One day when the teacher and Peter were returning to the school from the grocery store, Peter spotted a pair of cowboy boots in the window of the haberdashery, and they stopped to look. "Those are mighty pretty boots," Peter sighed.

"Would you like to make believe you were going to buy them? We could go in and ask the man if there is money enough in the box to buy them?"

Peter nodded, and the inquiry was made. "The boots are twelve dollars and ninety-five cents," said the storekeeper.

Peter looked at the storekeeper for a long time and then said, "Let's go back to school."

Once outside Peter said, "All people in stores want to do is take all your money."

Peter is still not convinced that storekeepers do not have boxes and boxes of money.

This is what the teacher hopes to accomplish with Peter next year:

Writing numbers to 500 or more.
It is hoped that when Peter reaches 201 with guidance, he can go on alone.

Learn the "hard" addition combinations.
This will have to be done with discs, flash cards, and other devices.

Learn as many subtraction facts as Peter can retain.
Here again discs, flash cards, abacus, and other devices will be employed.

Telling time at least at the hour and half hour.

The last ten weeks of the year, the teacher hopes Peter can learn a little about reading a ruler, and start working a little on oral problems.

EDITOR'S NOTE: Have you had a boy like Peter or one nearly down to his level in class? In many schools the Peters are not placed in special classes and this poses a very serious problem to a teacher who is responsible for perhaps thirty pupils. Note in Mrs. Vincent's account how very slowly Peter learned at first and how he gradually gained competence. The editor hopes that Peter's progress will be reported again in several years.

$$\begin{array}{r} \text{S E N D} \\ +\text{M O R E} \\ \hline \text{M O N E Y} \end{array}$$

Can you find numbers to substitute for the letters so this is true?

Which would you sooner have a pound of silver dollars or four pounds of silver quarters? Which is heavier a pound of gold or a pound of feathers?

Class Participation in a Relay Game

Kenneth P. Kidd
University of Florida, Gainesville

Experience in teaching arithmetic tells us that reliance upon one or two types of activities is not the most effective way to develop competence in arithmetic—whether the type of activity be drill, discussions of meanings of abstract arithmetic ideas, or solving problems. The arithmetic program in today's schools, we believe, should provide a variety of activities.

One of the activities which has been found to produce desirable results is a relay game in which the total class participates. This game has been designed to

(1) place a premium on the development of accuracy and independent work habits,
(2) involve all of the students all of the time, and
(3) provide exercises or problems of varying difficulty that might be appropriate for the different levels of competence of the students.

Procedures

Separate the class into two groups which shall be called A and B. Duplicate two similar lists of exercises and problems. Each list should have 20 items. One list should be labeled for the A group and the other for the B group. In one list the items should be designated by numbers 1 to 20 and in the other list by numbers 21 to 40. Each student should be given both lists.

This game is to be conducted like a relay. One particular piece of chalk should be used by members of group A and another piece (preferably of a different color) by members of group B. When the game has been started, only one person from a group may go to the blackboard and use that group's piece of chalk at any time.

Each person at his seat should be continuously engaged in

(1) detecting and correcting an opponent's error, for which his team will receive triple point value if he records same on blackboard,
(2) detecting and correcting a team mate's error to prevent an opponent from doing so and collecting triple point value,
(3) working exercises in his team's list which have not already been correctly worked.

It is suggested that a definite order be predetermined for having the members of each group go to the blackboard. In order to further encourage members to be on the alert to detect errors, opportunities should occasionally be given for anyone to go to the blackboard out of turn. One way to do this is to allow "Time for Correction," to be called for a group after five members from that group have gone to the blackboard in order. During this time opportunity may be given to any *one* person in that group to correct an error.

When a person goes to the blackboard, he should first write down problem number and then put down his solution. He cannot put up the correction to an opponent's error if one of the opponents is then at the blackboard working on the same problem.

If the items in the list involve complicated solutions, the relay will drag. If some of the less able students find that the easier problems are not left for them by their more able team mates, it may be wise to specify 1 point value for easy problems and 2 points for the more difficult ones.

Scoring

The game ends after a specified time or after all problems have been worked to the satisfaction of both teams. The team with the higher total score wins. A team's total score is determined by counting up the point values for *every* item worked correctly by members of that team remembering that the point value is tripled for each correction of an opponent's error.

EDITOR'S NOTE: This is a sample of one type of game device which may be used in teaching and learning arithmetic. Dramatic play is another type of activity which may have a "game" element. Such devices and activities serve not only to whet the interest of pupils but also the better games result in worthwhile learning. The factors of interest and participation are very important in learning at all levels. But learning arithmetic may not be automatic in a game, the wise teacher will note certain aspects that she may wish to stress in direct teaching. Many teachers have used the old-fashioned "spell down" as a review technique and as a more interesting type of practice or drill.

The Seattle Meeting

ELIZABETH ROUDEBUSH
Seattle, Washington

TEACHERS OF ARITHMETIC AND MATHEMATICS gathered at Seattle in August to discuss their mutual problems. This was the fourteenth summer meeting of the National Council of Teachers of Mathematics with the Puget Sound Council of Teachers of Mathematics as hosts. Several interesting papers in the field of arithmetic were presented and may become available for publication. Two discusions are reported below.

Professor Peter Spencer of Claremont College discussed some impacts of psychological theories on arithmetic. He pointed out that teaching demands that teachers study human behavior (psychology) because they are practicing psychologists in the classroom. They have developed theories of human nature which affect their performance. It was school teachers who showed Binet how to measure intelligence but it took a Binet to refine and develop an effective measuring instrument. So much of teachers' psychology is too closely related to the obvious but "beyond the obvious lies the truth." The learner needs to accompish what others already know and hence one who knows (a teacher) may simply tell the one who needs to learn. This simple theory leads to the "tell-and-do" procedure in teaching. This "tell-and-do" instruction is most common. Textbooks and instructional materials are largely adapted to it with the result that much education is mere habit formation (memorization). Teachers need to guard against these fixed ideas and to mechanistic theories of learning.

Many teachers desire their pupils to understand and appreciate the things they are to learn. Discovery, invention, and creative imagination are looked upon as necessary for developing meanings. The organismic and field theories of learning give support to these ideas. In modern psychology much is made of "projection." Education is fundamentally based upon projection of one's self and of what one has become into situations one faces.

It is most important that teachers' theories of learning and human behavior be valid and well thought out. Probably the most important phase in the development of a teacher is that of increasing her ability to sense, to understand, and to

utilize human behavior effectively.

Mrs. Marguerite Brydegaard of San Diego State College gave suggestions for enriching the teaching of arithmetic. She prescribed five educational vitamins that should lead to the following desirable results.

1. Exploration into the concepts underlying topics and processes developed in the elementary school. These explorations extend the concepts and their interpretation beyond the usual essentials. They lead to the *how* and the *why* and to deriving procedures that generate power for mathematical learning.
2. Explorations into applications that open new horizons for mathematical thinking —horizons that pupils and teachers explore together.
3. Much *re*-creational activity in which new ideas are created thru sensing, discovering, inventing, and formulating concepts. This does not refer to the recreational activities that are designed to offer play, amusement and refreshment but that may offer little in the way of stimulation of mathematical thinking.
4. Facility of expression should take wing with these vitamins.
5. A large measure of "common sense" operates in this program for enrichment. The perplexing questions of how to express answers, what to do with the remainder in division examples, etc., should be challenged with the "common sense" approach of concepts concerning what you are measuring, the purpose of the measurement, etc.

The dividends for such a program are rewarding. Out of it grows an educational climate and generation of power that is truly rewarding to teachers and to pupils.

Season's Greetings

BEN A. SUELTZ, *Editor*
ESTHER SWENSON, *Assoc. Editor*
MERLE AHRENDT, *Bus. Manager*

BOOK REVIEWS

Understanding Numbers: Their History and Use, Philip S. Jones, Ulrich's Bookstore, Ann Arbor, Michigan, 1954. 50 pages, paper, $1.00.

Here is a book containing seven lessons that Professor Jones originally presented as a television sequence. The lessons are precisely what the title suggests. They do not attempt to give a history of numbers or to tell how to teach arithmetic. Rather, here is a sequence for interested adults who want to be intelligent about numbers. True, Mr. Jones has given a good deal of history but he also has pointed out many modern uses of numbers. The book is illustrated. The large pages ($8\frac{1}{2}\times 11$) make this comparable with ordinary books of double the number of pages.

Who will find use for this book? It should appeal to teachers, to junior high school pupils, to high school pupils, and to curious adults. A few chapter headings suggest the scope and coverage: "The Earliest Numbers," "Big Numbers," "Fundamental Operations," and "New Numbers." Included are thirty-one projects ranging alphabetically from "Binary System and 'Mind Reading' " to "Unsolved Problems." Many of these topics are intriguing to intelligent adults.

BEN A. SUELTZ

I Work by Myself, Caroline Hatton Clark and Elizabeth Elsbree, World Book Co., 1954. 66 pages, paper, $0.48.

As the title implies this workbook aims to provide a sequence of learnings for first-graders. Children can work individually with a minimum of help from another person. Some schools will want to use this book for group activity and others will find it valuable for slow learners who need individual help from a teacher or parent. It is pictorial in presentation. Color enhances the interest for children. A good feature is a five-page guide for teachers in which the purposes of the activities with numbers are stated and procedures for use are explained. Teachers who want something to aid in building number concepts for their slow learners in grades two and three will find this book useful.

BEN A. SUELTZ

Two typists were hired by the same company and were given a choice of salary plans. The first girl chose a plan with an annual salary of $2400 and an increase of $240 per year for five years. The second girl chose a semi-annual salary of $1200 plus an increase of $60 every six months for five years. Which girl has the better plan?